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Pre-Calculus 11: HW Ch 1 Review

1. Find the next three terms in each sequence.

a. $-8, -5, -2, \cdots$

$-2 + 3 = 1$
$+3 \quad +3$
$1 + 3 = 4$
$3 + 4 = 7$
$\boxed{1, 4, 7}$

b. $\dfrac{3}{10}, \dfrac{4}{5}, \dfrac{13}{10}, \cdots$

$\dfrac{13}{10} + \dfrac{5}{10} = \dfrac{18}{10} = 1.8$

$0.5 \quad 0.5$

$1.8 + 0.5 = 2.3$
$2.3 + 0.5 = 2.8$

$\boxed{1.8, 2.3, 2.8}$

2. The general term of a sequence is given. Write the first 5 terms.

a. $t_n = 10 - 2n$

$10 - 2 \times 1 = 8$ $10 - 2 \times 5 = 0$
$10 - 2 \times 2 = 6$ $\boxed{8, 6, 4, 2, 0}$
$10 - 2 \times 3 = 4$
$10 - 2 \times 4 = 2$

b. $t_n = n^2 - 5$

$1^2 - 5 = -4$ $4^2 - 5 = 11$
$2^2 - 5 = -1$ $5^2 - 5 = 20$
$3^2 - 5 = 4$ $\boxed{-4, -1, 4, 11, 20}$

c. $t_n = \dfrac{n}{3n - 2}$

$\dfrac{3}{3 \times 3 - 2} = \dfrac{3}{7}$

$\dfrac{1}{3 \times 1 - 2} = 1$

$\dfrac{2}{3 \times 2 - 2} = 2$

$\dfrac{4}{3 \times 4 - 2} = \dfrac{2}{5}$

$\dfrac{5}{3 \times 5 - 2} = \dfrac{5}{13}$

$\boxed{1, 2, \dfrac{3}{7}, \dfrac{2}{5}, \dfrac{5}{13}}$

d. $t_n = 3(-4)^{n-1}$

$3(-4)^{1-1} = 3$ $3(-4)^{5-1} = 768$
$3(-4)^{2-1} = -12$
$3(-4)^{3-1} = 48$ $\boxed{3, -12, 48, -192, 768}$
$3(-4)^{4-1} = -192$

3. Find the number of terms in the arithmetic sequence:

a. $8, 12, 16, \cdots, 400$

$r = 4$ $x = $ The gap b/t the terms

$8 + 4x = 400$
$4x = 392$ $98 + 1 = 99$
$x = 98$ $\boxed{99}$

b. $-4, -2.6, -1.2, \cdots, 36.6$

$r = \dfrac{7}{5}$ $x = $ The gap b/t the terms

$-4 + \dfrac{7}{5}x = 36.6$ $29 + 1 = 30$
$\dfrac{7}{5}x = 40.6$ $\boxed{/30}$
$x = 29$

4. The 3 consecutive terms of an arithmetic sequence are 3.6, y, 8.2. Find the value of y.

$r = \dfrac{8.2 - 3.6}{2}$ $3.6 + 2.3 = 5.9$

$= 2.3$

$\boxed{\text{Ans: } 5.9}$

5. Find the difference between the 4th term and the 10th term of the arithmetic sequence $\{2, -3, -8, \ldots\}$.

$r = -5$

$10^{\text{th}} \text{ term} = 2 + (-5) \times 9 = -43$
$4^{\text{th}} \text{ term} = 2 + (-5) \times 3 = -13$

$-43 - (-13) = -30$

$\boxed{\text{Ans: } -30}$

6. The 10th term in an arithmetic sequence is 8x and the 4th term is −4x. Determine the first term a.

$$- - \underset{\smile}{-} \underset{\smile}{-} \overset{-4X}{-} \underset{\smile}{-} \underset{\smile}{-} \underset{\smile}{-} \underset{\smile}{-} \overset{8X}{-}$$

$$8X - (-4X) = 12X$$
$$12X \div 6 = 2X$$
$$r = 2X$$

$$a + 2X(3) = -4X$$
$$a + 6X = -4X$$
$$a = -10X$$

$\boxed{Ans = -10X}$

7. Find the first 3 terms of the arithmetic sequence with third term 4 and sixth term −29.

$$- - \overset{4}{-} \underset{\smile}{-} \underset{\smile}{-} \underset{\smile}{-} \overset{-29}{-}$$

$$(-29 - 4) \div 3 = -11$$
$$r = -11$$

$$4 + 11 = 15$$
$$15 + 11 = 26$$

$\boxed{Ans: 26, 15, 4}$

8. Find the sum of the arithmetic series: $(-300) + (-297) + (-294) + \cdots + 309$

$-300 \sim 300$ cancel out

$$309 + 306 + 303 = 918$$

$\boxed{Ans : 918}$

9. You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 6 seconds? $r = 32$

$$16, 48, 80$$

$$16 + 32 \times 5 = 176$$
$$\uparrow$$
$$(6-1)$$

$\boxed{Ans: 176}$

10. In an arithmetic sequence, the 11th term is 53 and the sum of the 5th and 7th terms is 56. Find the first 3 terms.

$$a \quad a+r \quad a+2r \quad \cdots \quad 53$$

$$a + 10r = 53$$

$$a + 4r + a + 6r = 56$$
$$2a + 10r = 56$$

$$2a + 10r = 56$$
$$-) \ a + 10r = 53$$
$$\overline{}$$
$$a = 3$$
$$r = 5$$

$$3 + 5 = 8$$
$$8 + 5 = 13$$

$\boxed{Ans: 3, 8, 13}$

11. The sum of the first 2 terms of an arithmetic sequence is 15, and the sum of the next 2 terms is 43. Find the first 3 terms.

$$a + a + r = 15 \longrightarrow 2a + r = 15$$
$$a + 2r + a + 3r = 43 \rightarrow 2a + 5r = 43$$

$$2a + 5r = 43$$
$$-) \ 2a + r = 15$$
$$\overline{}$$
$$4r = 28$$
$$r = 7$$
$$a = 4$$

$$4 + 7 = 11$$
$$11 + 7 = 18$$

$\boxed{4, 11, 18}$

12. If 5 + x, 8, and 1 + 2x are consecutive terms in an arithmetic sequence, find x.

change sign

$$8 \underline{-(5 + X)} = 1 + 2X - 8$$
$$8 - 5 - X = 1 + 2X - 8$$
$$10 = 3X$$
$$X = \frac{10}{3}$$

r

$\boxed{Ans: \frac{10}{3}}$

13. Find the next three terms in each sequence.

a. $2, +4, 6, \cdots$

$+2 \quad +2$

$6 + 2 = 8$

$8 + 2 = 10$

$10 + 2 = 12$

Ans = 8, 10, 12

b. $x, -2x^2, 4x^3, \cdots$

$-2x \quad -2x$

$4x^3 \times (-2x) = -8x^4$

$-8x^4 \times (-2x) = 16x^5$

$16x^5 \times (-2x) = -32x^6$

$-8x^4, 16x^5, -32x^6$

14. If the second term of a geometric sequence is 14 and the fifth term is 112, find the first 3 terms.

$\_\_ 14 \_\_ \_\_ 112$

$14 \times 3d = 112$

$3d = 8 \quad d = \frac{8}{3}$

$14 \div \frac{8}{3} = \frac{-21}{4}$

$14 \times \frac{8}{3} = \frac{112}{3}$

Ans: $\frac{21}{4}, 8, \frac{112}{3}$

15. If x - 3, x + 1, and 4x - 2 are consecutive terms in a geometric sequence, find x.

$\frac{x+1}{x-3} \neq \frac{4x-2}{x+1}$

$(x+1)(x+1) = (4x-2)(x-3)$

$x^2 + 2x + 1 = 4x^2 - 12x - 2x + 6$

$3x^2 - 16x + 5 = 0$

$1 \quad \times \quad 5$

$3 \quad \times \quad 1$

$(x+5)(3x+1) = 0$

$x = -5, \frac{-1}{3}$

16. In a geometric sequence, $t_3 + t_4 = 36$, and $t_4 + t_5 = 108$; find the first 5 terms of the sequence.

$ar^2(1+r) = 36 \quad ar^2 \quad ar^3 \quad ar^3 \quad ar^4$

$ar^3(1+r) = 108$

$108 \div 36 = 3 \cdots r$

$a \Rightarrow 9a + 27a = 36 \quad a = 1$

$1, 3, 9, 27, 81$

17. Sixty-four players are entered in a tennis tournament. When a player loses a match, he or she drops out; the winners go on to the next round. What is the total number of matches that must be played before a winner is decided?

You need to eliminate 63 players in order to get a winner, so 63 games need to be played
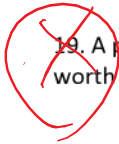
Ans: 63

18. Find Sn, the sum of the first n terms, for the following geometric series. $a = 1944$, $t_4 = 1$, and $r = \frac{-1}{4}$

$\frac{1944}{1 + (\frac{-1}{4})} = 1555.2$

Infinite Geo $\Rightarrow -1 < r < 1$

$-1 < \frac{-1}{4}$

Ans: 1555.2

19. A person invests $900 at a fixed rate of 5.4% per annum, compounded every month. How much is the investment worth after 10 years?

(crossed out)

20. A painting is purchased for $1.7 million and 5 years later sold for $2.6 million. What yearly rate has the painting appreciated by?

1.7 ———————————————— 2.6

1y   2y   3y   4y   5y

$2.6 \div 1.7 = \frac{26}{17}$ $\frac{26}{17} \div 5 = \frac{26}{85} = 0.30588 \doteq 31\%$

Ans: 31%

21. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

$24 \div 2 = 12$

500   1000   2000   4000

2h   2h

$500 \times 2^{12} = 2048000$

Ans: 2048000

22. Find the next three terms in each sequence.

a. $\frac{2}{3}, \frac{-1}{3}, \frac{1}{6}, \cdots$ $\frac{1}{6} \times \left(\frac{-1}{2}\right) = \frac{-1}{12} - T_4$

$\frac{-1}{2}$ $\frac{-1}{2}$ $\frac{-1}{12} \times \left(\frac{-1}{2}\right) = \frac{1}{24} - T_5$

$\frac{1}{24} \times \left(\frac{-1}{2}\right) = \frac{-1}{48} - T_6$

b. $\frac{c}{d}, -\frac{c^2}{d^3}, \frac{c^3}{d^5}, \cdots$ $\frac{c^3}{d^5} \times \left(\frac{-c}{d^2}\right) = \frac{-c^4}{d^7} - T_4$

$\frac{-c}{d^2}$ $\frac{-c}{d^2}$ $\frac{-c^4}{d^7} \times \left(\frac{-c}{d^2}\right) = \frac{c^5}{d^9} - T_5$

$\frac{c^5}{d^9} \times \left(\frac{-c}{d^2}\right) = \frac{-c^6}{d^{11}} - T_6$

23. The sum of an infinite geometric series is $\frac{24}{7}$ and the common ratio is $-\frac{3}{4}$. Find the first term.

$\frac{a}{1 - \left(-\frac{3}{4}\right)} = \frac{24}{7}$

$\frac{a}{\frac{7}{4}} = \frac{24}{7}$ $7a = 24 \times \frac{7}{4}$

$a = 6$

Ans = 6

24. Given the terms $t_{10} = \frac{3}{512}$ and $t_{15} = \frac{3}{16384}$ of a geometric sequence, find the exact value of the term t_{30} of the sequence.

$\frac{3}{512}$ ———————— $\frac{3}{16384}$

$\frac{3}{512} \times X^5 = \frac{3}{16384}$

$X^5 = \frac{1}{32}$

$X = \frac{1}{2}$

$\frac{3}{16384} \times \left(\frac{1}{2}\right)^{14} = 1.1175870 9 \times 10^{-3}$

25. A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half life of 1 day. Find the amount of radioactive material in the sample at the beginning of the 7th day.

500 250

$\frac{1}{2}$

$500 \times \left(\frac{1}{2}\right)^6 = 7.8125$

$\boxed{Ans = 7.8125 \, mg}$

26. A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. Calculate the total vertical distance the ball travels before coming to rest. $\to 0$

8 , 6.4 , $\underset{0.8}{} \; \underset{0.8}{}$, 0

$\dfrac{8 - 0.8 \times 0}{1 - 0.8} = 40$

$\boxed{Ans = 40}$

27. Evaluate:

a. $\displaystyle\sum_{i=1}^{75}(2i+3)$ Arithmetic

$\underset{5}{2(1)+3} + \underset{7}{2(2)+3} + \underset{9}{2(3)+3} \cdots \underset{153}{2(75)+3}$

$\dfrac{(5+153)75}{2} = 5925$

$\boxed{Ans = 5925}$

b. $\displaystyle\sum_{i=12}^{87}(4i-1)$ Arithmetic

$\underset{47}{(4\times12-1)} + \underset{51}{(4\times13-1)} + \underset{55}{(4\times N-1)} \cdots \underset{347}{(4\times87-1)}$

← Number of term

$\dfrac{(47+347)(87-12+1)}{2} = \boxed{14972}$

c. $\displaystyle\sum_{i=3}^{27}(2)^{2i-3}$ Geometric

$\dfrac{(2)^{2(3)-3}}{2^3=8} + \underset{2^5=32}{(2)^{2(4)-3}} + \underset{2^7=128}{2^{2(5)-3}} \cdots 2^{2(27)-3}$

$\dfrac{2^3 - 4 \times 2^{51}}{1-4} = \dfrac{2^3 - 2^2 \times 2^{51}}{-3}$

$= \boxed{\dfrac{2^3 - 2^{53}}{-3}}$

$= 3.002399752 \times 10^{15}$

b. $\displaystyle\sum_{i=1}^{\infty}4\left(\frac{2}{5}\right)^{i-1}$ Infinite Geometric

$\underset{4}{4\left(\frac{2}{5}\right)^{1-1}} + \underset{\frac{8}{5}}{4\left(\frac{2}{5}\right)^{2-1}} \cdots \cdot 4\left(\frac{2}{5}\right)^{\infty-1}$

$d = \dfrac{2}{5}$

$\dfrac{4}{1-\frac{2}{5}} = \dfrac{20}{3}$

$\boxed{Ans = \dfrac{20}{3}}$